shifters using a segregated mode of single crystal YIG give most encouraging performance even in its crudest form. Performance can, no doubt, be improved with the proper design of microwave circuit, and, in the case of the composite resonator, that of the resonator. It should be pointed out that the segregated mode possesses great potentials in device applications such as filters, paramps, directional couplers, isolators, etc., particularly in the high power realm. The reasons are:

1) Previously, the threshold power for the excitation of spurious responses dictates the useful power level of the device. Now mode segregation from the manifold eliminates excitation of these undesirable responses, as evidenced by our high power tests. Therefore, power level can be greatly increased.
2) Sample size employed for the present single crystal YIG devices has been limited to a minimum because it has been observed that larger sizes give more spurious responses (Walker modes). With the mode-segregation effects, this problem is eliminated and much higher power capacity is now possible since the Q of the material is high.

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Comments on "A Large Signal Analysis Leading to Intermodulation Distortion Prediction in Abrupt Junction Varactor Upconverters"

In the above paper by Perlow and Perlman,[1] the authors use steps that are not mathematically correct and which lead to incorrect results. The authors write the matrix equations for the three-frequency upconverter [(1) and (2)] relating the Fourier coefficients i_1, i_2, and i_3 to the Fourier coefficients of the pump and signal voltages. They then derive an expression for the gain G_i as a function of i_1 the Fourier coefficient of the signal current [(10)]. After expanding G_i in a power series [(12)] they substitute a time function [(13)] for the Fourier coefficient i_1 which is clearly not permissible. They further substitute for i_1 the sum of two sinusoids although (1) and (2) are valid for only a single sinusoid at the input.

That the results are incorrect can be seen as follows. If one drives, but not overdrives, an abrupt junction varactor with frequencies f_{s1}, f_{s2}, and f_p, where f_{s1} and f_{s2} are two input signals whose frequency separation is small and hence fall in the input bandwidth, voltages are generated across the diode at $f_p \pm f_{s1}$, $f_p \pm f_{s2}$, $f_{s1} \pm f_{s2}$, $2f_{s1}$, $2f_{s2}$, and $2f_p$. If the diode sees an open circuit at all frequencies except f_p and narrow bands of frequencies about the input and output frequencies then the only currents that flow due to these voltages are f_p+f_{s1} and f_p+f_{s2}. These currents in turn mix with f_{s1}, f_{s2}, f_p as well as doubling to produce voltages at $2(f_p+f_{s1})$, $2(f_p+f_{s2})$, $2f_p+f_{s2}+f_{s1}$, $f_p+f_{s2}\pm f_{s1}$, f_p+2f_{s2}, $2f_p+f_{s2}$, $f_p+f_{s1}\pm f_{s2}$, f_p+2f_{s1}, $2f_p+f_{s1}$. If, as assumed above, the varactor sees an open circuit at all of these frequencies, currents do not flow at these frequencies and no intermodulation is produced even though the gain of the upconverter is a function of drive level. If on the other hand, for example, the impedance seen by the varactor at $f_{s1}-f_{s2}$ is small, current will flow at this frequency which will then mix with f_{s1} to produce a voltage at $2f_{s1}-f_{s2}$ which lies in the input bandwidth. The current at this frequency can then mix with f_p to produce an intermodulation product at $f_p+2f_{s1}-f_{s2}$ which will appear in the output. We thus see that the level of intermodulation is a function of the "out-of-band" impedances and cannot be determined without a knowledge of them. The results derived by Perlow and Perlman are independent of these impedances and hence cannot be correct.

It is interesting to note that if frequencies $f_p+k(f_{s2}-f_{s1})$ can flow in the pump circuit for all k less than n then intermodulation products $f_p+f_{s1}+k(f_{s1}-f_{s2})$ and $f_p-f_{s1}+k(f_{s1}-f_{s2})$ will appear in the output for all k less than n. These currents flowing in the pump circuit as well as currents at $m(f_{s1}-f_{s2})$ flowing in the bias circuit are probably the major cause of intermodulation products in the output. If one overdrives the varactor, however, or uses a graded junction, then higher-order intermodulation products can be generated even in the absence of idlers.

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[1] S. M. Perlow and B. S. Perlman, *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-13, pp 820–827, November 1965.

S. M. Perlow and B. S. Perlman[2]

Grayzel's conclusion that the mathematics of the paper are not correct seems to be based upon a misinterpretation of the meaning of (10).[1] Equation (10) represents the dynamic transfer characteristic of the upconverter, where i_1 and i_3 represent the instantaneous values of the input and output currents. This transfer characteristic is nonlinear. As the input current is continually increased, the output current finally reaches a certain level which cannot be exceeded. A sinusoidal input current of sufficient amplitude will therefore drive the upconverter into the nonlinear gain region, thereby producing an output rich in spurious content. This transfer characteristic was treated as any other nonlinear transfer characteristic would be analyzed. That is, it was expanded into a Taylor series with sinusoidal inputs. For example, the ΔC vs. V characteristic is not derived using sinusoidal voltage, but when it is analyzed for spurious content, sinusoids are used.

This method of utilizing the upconverter characteristic rather than the diode characteristic was undertaken because of anomalous results for abrupt junction varactors. Using the abrupt junction diode characteristic to predict intermodulation distortion results in an erroneous conclusion. That is, there is no intermodulation distortion. This result is obtained for any square-law diode. However, when these square-law diodes are placed in upconverters or mixers, intermodulation distortion occurs and is due to the nonlinear transfer of the complete device rather than the diode itself. Therefore, an analysis of the complete device's transfer characteristic must be performed to obtain the amplitudes of the intermodulation distortion products.

Grayzel's second criticism is that the level of the intermodulation is a function of the "out-of-band" impedances and cannot be determined without a knowledge of them. He also makes the assumption that the diode sees an open circuit at all frequencies except f_p and narrow bands of frequencies about the input and output frequencies. He says, using this assumption, that no intermodulation is produced even though the gain of the upconverter is a function of drive level.

These assumptions were never made by us. In fact, compensation for any reduced bandwidth is shown in (39) of the paper.

The only assumption made was that the signal, pump, and output circuits were tuned to their resonant frequencies. No mention of circuit G and bandwidth was made. In the intermodulation analysis, only the terms in the expansion that contributed to the intermodulation distortion were used. This implicitly assumes that f_{s1}, f_{s2}, $2f_{s1}$, $2f_{s2}$, $3f_{s1}$, $3f_{s2}$, $2f_{s1}-f_{s2}$, $3f_{s1}-2f_{s2}$, $4f_{s1}-3f_{s2}$, etc., flowed in the signal circuit at full amplitudes. Similarly, $f_p\pm f_s$, $f_p\pm f_{s2}$, $f_p\pm 2f_{s1}-f_{s2}$, $f_p\pm 3f_{s1}-2f_{s2}$, etc., flowed in the output circuit at their full amplitudes also.

Grayzel makes objections to the analysis on several points and on each point he states that our conclusions are incorrect. However, this is not a purely theoretical result that cannot be experimentally verified. Indeed, the experimental results for the parametric upconverter bear these results out. In fact, (36) of the paper[1] has not only been experimentally verified for a parametric upconverter but also for the resistive mixer and even for maser amplifiers. It should be pointed out that these experimental results not only come from our own laboratories but also from measurements made by other companies.

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[2] Manuscript received May 16, 1966.

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There are mathematical inconsistencies in Perlow and Perlman's paper.[1] First, (10) is derived from the relationships of (1) and (2). Equations (1) and (2) are valid for a three-frequency upconverter, i.e., only currents at ω_1, ω_2, and ω_3 flow through the diode. The authors then introduce additional frequencies into (10) and, in fact, claim in their rebuttal that all idler frequencies flow at "full amplitude." Secondly, (1) and (2) are matrix equations relating the Fourier coefficients of the voltages to the Fourier coefficients of the currents. i_1, i_2, and i_3 are defined by the relationships preceding (1) and are not functions

[3] Manuscript received June 8, 1966.

of time. The authors, however, set i_1 equal to a time function in (10) and claim that i_1 and i_3 are time functions in their rebuttal.

Whether the curves presented by Perlow and Perlman fit measurements taken on a particular upconverter are not at issue here, only the correctness and validity of the derivations.

S. M. Perlow and B. S. Perlman[4]

As we have previously stated, (10) represents the transfer characteristic of the upconverter. As such, it is treated as any other nonlinear characteristic would be. Grayzel believes this to be mathematically inconsistent and leads to incorrect results. It will now be shown that two tones applied to the input may indeed be analyzed in such a manner.

When a two-tone test is used to measure intermodulation distortion, two signals of frequencies f_a and f_b are applied to the input. The output contains responses at every sum and difference frequency. The frequencies of interest for intermodulation distortion specifications are those corresponding to $(m+1)f_a - mf_b$ and $(m+1)f_b - mf_a$. This was the notation used in the paper. Equation (10) was expanded in a series and i_1 was replaced by cos $\omega_s t$ and then later on by cos $\omega_a t$+cos $\omega_b t$, expanded to all odd powers. The validity of this series may be seen quite easily by considering a different means of generating the two tones.

Let the two-tone input signals be separated in frequency by $2\omega_\epsilon$. Therefore

$$\omega_a = \omega_1 + \omega_\epsilon$$
$$\omega_b = \omega_1 - \omega_\epsilon$$

and (22) becomes:

$$\begin{aligned} i_{\text{signal}} &= |i| (\cos \omega_a t + \cos \omega_b t) \\ &= |i| [\cos (\omega_1 + \omega_\epsilon)t \\ &\quad + \cos (\omega_1 - \omega_\epsilon)t] \\ &= 2|i| \cos \omega_\epsilon t \cos \omega_1 t. \end{aligned}$$

That is, the two original tones may be replaced by a single DSBSC tone. The magnitude of the input signal is no longer a constant but now becomes $2[i]$ cos $\omega_\epsilon t$, where $\omega_\epsilon \ll \omega_s$. Equation (13) may now be written as

$$\begin{aligned} i_3 = A_0|i| [&\hat{\imath}_1 - |A_1 i_1|^2 \hat{\imath}^3 + |A_1 i_1|^4 \hat{\imath}^5 \\ &- |A_1 i_1|^6 \hat{\imath}^7 + \cdots] \end{aligned}$$

where

$$\hat{\imath}_1 = \cos \omega_\epsilon t.$$

The intermodulation frequencies corresponding to $(m+1)f_a - mf_b$ and $(m+1)f_b - mf_a$ now become $f_1 \pm (2m+1)f_\epsilon$. Note that the intermodulation frequencies are harmonically related to the difference in frequency between ω_a and ω_b.

The second term of the above expansion gives rise to the desired output plus a contribution to the first intermodulation distortion product term of the form $K \cos (\omega_s + 3\omega_\epsilon)t$, or if ω_a and ω_b are used, $K \cos (2\omega_a - \omega_b)t$. The coefficients of this expansion are exactly the same as the coefficients of the expansion in the paper, and the intermodulation frequencies are the same. The equality of the two expansions is thus proven.

This approach, although slightly more complicated conceptually, is mathematically rigorous. It takes into account all of Grayzel's objections of mathematical inconsistencies and leads to the same expansion found in (21) and (22) of the paper, thereby showing the correctness and validity of the results.

[4] Manuscript received June 22, 1966.

On Changing the Coupling into a Microwave Cavity by Means of a Stub Tuner

In microwave measurements involving a resonant cavity it is sometimes desirable to be able to make a continuous adjustment of the coupling coefficient, defined as[1]

$$\beta = \frac{Q_0}{Q_{\text{ext}}}. \tag{1}$$

Variation of Q_0 by adjustment of the spatial distribution of loss within the cavity and variation of Q_{ext} by mechanical adjustment of the coupling loop or aperture are both often impractical, particularly in situations where it is difficult to gain access to the cavity while data are being taken. However, one can also vary the coupling coefficient by inserting a shunt susceptance (stub tuner) into the feeder line, at some distance away from the cavity. The problem of calculating the magnitude and location of this susceptance differs somewhat from the standard fixed-frequency or narrow-band matching problem,[2] because the impedance being "matched" has a strong and characteristic frequency dependence. The purpose of this correspondence is to consider this design problem.

By attaching the shunt susceptance, one forms in effect a new composite resonant system (Fig. 1), consisting of the shunt susceptance added to the cavity admittance, with the latter transformed by the line segment between the detuned-short (DS) position and the position of the shunt susceptance. The DS position serves as a convenient reference location because the equivalent representation of the cavity assumes the simple shunt form of Fig. 1 only when viewed from there. The Q of this composite system can in principle be calculated analytically from the known composite admittance function,[3] but a much simpler approach can be made by using the Smith chart.

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[1] E. L. Ginzton, *Microwave Measurements*. New York: McGraw-Hill, 1957, ch. 9.

[2] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw-Hill, 1966, ch. 5.

[3] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits*. New York: McGraw-Hill, 1948, ch. 7.

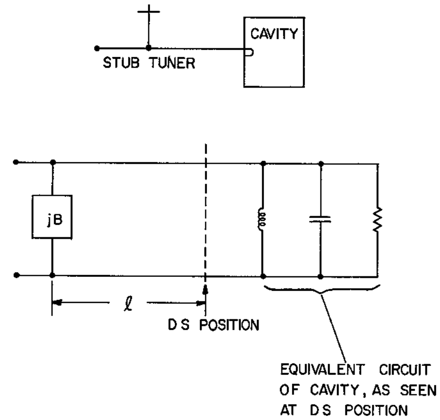

Fig. 1. Composite resonator formed by adding a shunt susceptance in the cavity feeder line.

On a Smith chart a plot of the cavity impedance, observed at the DS position over a frequency range near resonance, will result in a circle through the point $R/Z_0 = 0$. The numerical value of β can then be read directly from this characteristic as the normalized resistance offered by the cavity at resonance. The diameter of the impedance diagram is thus directly related to the magnitude of the coupling coefficient β. Actually, for the purposes of this discussion, the cavity *admittance* circle (as seen at the DS position) is of greater interest; it is obtained by reflecting the impedance circle through the center of the Smith chart.

In using the Smith chart to describe the behavior of the composite system one assumes, first, that the shunt susceptance is essentially constant with frequency over the entire cavity bandwidth. This assumption is quite realistic, provided the magnitude of the shunt susceptance is kept reasonably small ($B/Y_0 < 1$). Second, the transmission line segment between shunt susceptance and cavity must not have a large standing-wave field and it must be reasonably short, so that one can assume the amount of energy stored there to be negligible compared to the energy stored in the cavity. This assumption also is satisfied only for small values of shunt susceptance. As the latter is increased, the Q_0 of the composite system will deviate more and more from the true Q_0 of the cavity.

We will now consider the behavior of the composite system. If a shunt susceptance is added at a position which is an integral number of half-wavelengths away from the DS ($l = (n/2)\lambda g$), the cavity will simply be detuned, since one is then adding a reactive element in parallel to the equivalent resonant circuit. The size of the admittance circle remains invariant in this case, and there is no resultant change in the coupling coefficient. If, on the other hand, a shunt susceptance ($\pm jB$) is added at DS$\pm\frac{1}{4}\lambda_g$, then the size of the circle must always decrease. This can be seen in Fig. 2, which shows the admittance circle y_1 rotated through $\frac{1}{4}\lambda_g$ to position y_2, where the shunt susceptance is then added. For example, if $B/Y_0 = +3.0$, the composite admittance circle then appears at y_3, and it is much smaller than y_1. Finally, from Fig. 2 it is clear that with the shunt susceptance placed near